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Poking at the correspondence principle with derivative couplings

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Abstract

It is shown that quantum mechanical systems obtained from a heuristic path integral quantization of lagrangians of the form

$$L = \sum_n \frac{1}{n!} f_n(q) \dot{q}^n$$

violate in general the correspondence principle, unless L is not more than quadratic in $\dot{q}$. The field theoretic counterpart of this result and its consequence on models of interacting scalar and vector fields and particularly the electroweak interactions, are briefly discussed.

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1 Introduction

In the early days of the old quantum theory, the correspondence principle played an essential role in finding the right theoretical models that describe the atomic systems [1]. Later on, the final formulation of quantum mechanics had to implement this principle automatically in the quantization program. This is realized through the more than familiar substitution rules :

$$\begin{aligned} q &\rightarrow Q \\ p &\rightarrow \frac{\hbar}{i} P \end{aligned} \tag{1}$$

in the classical hamiltonian, with the precaution that one should symmetrize properly in the operators Q and P in order to construct a hermitian hamiltonian. Equivalently one can use the path integral formulation to get the transition amplitudes [2,3],

$$< q; t \mid q'; t' >= \mathcal{N} \int [dp][dq] \mathrm{e}^{\frac{i}{\hbar} \int_{t'}^{t} dt \, (p\dot{q} - H[p,q])} \tag{2}$$

with a choice of quantization of the form

$$< q' \mid H(P,Q) \mid q >= \int \frac{dp}{2\pi\hbar} \mathrm{e}^{i\,p\,(q-q')/\hbar} \; H(p, \mathcal{S}(q,q')) \tag{3}$$

where $\mathcal{S}$ is any symmetric function verifying $\mathcal{S}(q,q) = q$. The usual choice $\mathcal{S}(q,q') = (q+q')/2$ is the so–called Wigner quantization. Equation (3) guarantees the hermiticity of the quantum hamiltonian as well as the correct continuum limit [3]. Here q and p denote respectively the quantum mechanical variables and their conjugate momenta.

Relying on eq.(2) we will prove in the next section, that the rule in eq.(1) together with eq.(3) can be insufficient to ensure a *correspondence* between the classical and quantum systems in the limit $\hbar \rightarrow 0$, and that this correspondence should be viewed as intimately related to a specific class of interactions[1]. The result generalizes easily to scalar field theory (and with some more work, to vector fields), and thus selects a

[1]For definiteness, subsequent use of eq.[2] will be made in the euclidean metric ($i \rightarrow -1$).

particular class of universally allowed effective lagrangians. Perhaps more importantly, it thus universally rejects a wide class of interactions. In section 3 we comment briefly the result as well as its physical meaning and possible implications, specifically in the case of electroweak interactions.

2 The proof

In this section we give the main steps which constitute our proof of the relevant constraints, while more technical details will be displayed elsewhere. We should however stress here that since the proof requires manipulation of formal series, one should be very careful that such a manipulation is not wildly biasing the result.

We start from a general classical lagrangian of the form

$$L = \sum_{n\geq 0} \frac{1}{n!} f_n(q) \dot{q}^n \tag{4}$$

where the dots denote time derivatives and the sum can be infinite. We will then have to determine the classical hamiltonian and subsequently integrate over the conjugate momenta p in eq.(2) to obtain the effective action S_{eff}. It will turn out that S_{eff} will generally develop, in the limit $\hbar \to 0$, an induced classical term which was not present in the initial classical lagrangian eq.(4). This inconsistency is avoided only if $n \leq 2$. The latter condition will constitute the selection criterion for the *allowed* interactions.

The first difficulty one has to face in deriving the explicit form of the classical hamiltonian

$$H(p,q) = p\dot{q} - L(q,\dot{q}) \tag{5}$$

is obviously due to the relation between $\dot{q}$ and the conjugate momentum p, namely

$$p = \sum_{n\geq 0} \frac{1}{n!} f_{n+1}(q) \dot{q}^n \tag{6}$$

Indeed one needs to invert eq.(6) to get $\dot{q} = \dot{q}(p)$, which seems a priori a desperately ugly task! Let us however write down a heuristic solution by inverting eq.(6) through a formal iterative procedure. One gets after a straightforward inspection

$$\dot{q} = -\frac{1}{f_2} [f_1 - p + \overset{\infty}{\underset{i=0}{\circ}} F(\sum_{n\geq 2} (-1)^n \frac{1}{n!} [(f_1 - p)/f_2]^n f_{n+1})] \tag{7}$$

where the function F is defined by

$$F(y) = \sum_{n\geq 2} (-1)^n \frac{f_{n+1}}{f_2^n} \sum_{p=0}^{n} \left(\begin{array}{c} n \\ p \end{array} \right) f_1^p \, y^{n-p}$$

with $f_n \equiv f_n(q)$ and $(\overset{\infty}{\underset{i=0}{\circ}} F)$ stands for an iterative application of F, *i.e.* $F \circ F \circ F \cdots \circ F \cdots$.

Upon use of eqs.(5–7) one then gets

$$H[p,q] = -\frac{p}{f_2}\mathcal{H} - \sum_{n\geq 0} \frac{1}{n!}(-1)^n \frac{f_n}{f_2^n}\,\mathcal{H}^n. \tag{8}$$

where

$$\mathcal{H} = f_1 - p + \overset{\infty}{\underset{i=0}{\circ}} F(\ \sum_{m\geq 2}(p)\ )$$

and

$$\sum_{m\geq 2}(p) \equiv \sum_{m\geq 2} (-1)^m \frac{1}{m!}\,[(f_1 - p)/f_2\,]^m\, f_{m+1}$$

$H(p,q)$ is thus obtained as a formal series in powers of p. Furthermore, injecting eq.(8) in eq.(2) and integrating out the conjugate momentum p, one obtains the complete effective lagrangian L^{eff} of the system under considertion. It is then possible to check whether the *correspondence principle* is satisfied, by comparing L in eq.(4) to the behaviour of L_{eff} in the limit $\hbar \to o$. Now to integrate out p explicitly is obvioulsy the next hurdle to face. For this we make use of the familiar Wick expansions, however in our case one will be able to exponentiate essential parts of the expansions, which will be sufficient to draw definite conclusions. Actually these parts have the same structure as those which are usually absent for instance in the Green's functions, as they vanish when the external source coupled linearly to the system is turned off. In the present case they are due to the linear term in p in the exponential, eq.(2), and are evidently non–vanishing. To illustrate what is going on in a rather simple way let us first retain only up to cubic terms and write down, to fix the notation,

$$p\dot{q} - H(p,q) = a_0 + b\,p - \frac{1}{2}a\ p^2 + \frac{1}{3!}c\,p^3 + \cdots \tag{9}$$

One then expands the exponential $e^{\frac{1}{3!}c\,p^3}$ in eq.(2) and integrates over p for each term of the expansion using the usual trick of successive differentiation with respect

$b/\hbar$. It is interesting to note that the leading terms in powers of b which are leading in the $\hbar$ expansion as well, have no symmetry factors and can thus be completely re–exponentiated. Including all other terms one is lead to the following result

$$\int dp\, \mathrm{e}^{\frac{-1}{\hbar}(a_0 + b\,p - \frac{1}{2}a\; p^2 + \frac{1}{3!}c\,p^3)} = \frac{\mathcal{N}}{\sqrt{Det[a]}}\mathrm{e}^{\frac{-1}{\hbar}(a_0 + \frac{1}{2}b\,a^{-1}\,b + \frac{1}{3!}c\,(a^{-1}\,b)^3 + \hbar\,\Delta^{(3)}(\hbar))} \tag{10}$$

where

$$\Delta^{(3)}(\hbar) = -ln[1+\mathrm{e}^{\frac{1}{\hbar}\frac{1}{3!}c\,(a^{-1}\,b)^3}\sum_{m\geq 1}\sum_{p\geq 1}^{[3m/2]}\frac{(-1)^{p-m}}{m!}(\frac{1}{3!}c)^m\,\hbar^{p-m}\,(a^{-1})^p(a^{-1}b)^{3m-2p}\;s^p_m] \tag{11}$$

and $s^p_m = \frac{(3m)!}{2^p\,p!\,(3m-2p)!}$ is a symmetry factor [2]. Comparison of eqs.(8) and(9) gives after the change of variable $p \rightarrow p - f_1$,

$$\begin{aligned} a_0 &= f_0(q) + f_1(q)\,\dot{q} \\ b &= \dot{q} \\ a &= \frac{1}{f_2(q)} \\ c &= \frac{f_3(q)}{f_2(q)^3} \end{aligned} \tag{12}$$

Inserting these values in eq.(10) yields finally the effective lagrangian up to third order

$$L^{(3)}_{eff} = f_0(q) + f_1(q)\dot{q} + \frac{1}{2}f_2(q)\dot{q}^2 + \frac{1}{3!}f_3(q)\dot{q}^3 + \hbar\Delta^{(3)}(\hbar) \tag{13}$$

One can actually obtain the full effective lagrangian iteratively along the same lines, using as expansion parameter $\epsilon = 1/f_2(q)$, where it is assumed that $f_n(q)$ is of order one, if $n \geq 3$. This expansion is of course nothing but a perturbation around the gaussian form. The general form of L_{eff} to the N^{th} order is found to be

$$L^{(N)}_{eff} = \sum_{n=0}^{N}\frac{1}{n!}f_n(q)\dot{q}^n + \hbar\Delta(\hbar) \tag{14}$$

[2] Although not clearly explicited throughout, one should keep in mind that $f_n(q)$ and q can in general be respectively a tensor of rank n and a vector, in a given space of degrees of freedom. Thus from eq.(12), a_0 is a pure number, b a vector, $a = [f_2(q)]^{-1}$ a matrix and $c = f_3^{lmn}a_{li}a_{mj}a_{nk}$ a tensor of rank three. Also all the products in the exponential in eq.(10) should be understood in the functional sense with the integration $\int dt$.

where $\Delta(\hbar)$ is now a much more complicated expression than in eq.(11) and will not be displayed here. Again eq.(14) can be obtainded by resumming the leading powers in b similarly to eq.(10), and shows clearly that one does recover the classical lagrangian L as part of the effective largrangian (as it should). However, the question we want to address now is whether L is the full classical part of the effective lagrangian, that is whether $\hbar\Delta(\hbar)$ vanishes in the limit $\hbar \to 0$. The answer to this question requires technical but rather straightforward manipulations. For the sake of simplicity we present this here only in the case where $\Delta(\hbar)$ is given by eq.(11). Note that in eq.(11) one cannot obtain directly the behaviour of $\Delta(\hbar)$ in the classical limit, since in the argument of the log the exponential increase can be possibly compensated by the *infinite* alternating power series in $1/\hbar$. We will actually prove that, as far as $c \neq 0$ the argument of the log behaves like $\mathrm{e}^{\frac{-1}{\hbar}}$ in the quasistatic limit ($\dot{q}$ very small) that is

$\hbar\Delta(\hbar)$ *induces extra classical effects which are not present in the initial classical lagrangian, unless* $c = 0$.

As the proof is mainly technical we exhibit here only the important steps. The following identities will prove useful :

$$\frac{(3m)!}{(3m-2p)!} = \sum_{q=0}^{2p-1} \frac{(-1)^q}{q!} (3m)^{(2p-q)} \sum_{i_1 \neq i_2 \neq i_3 \cdots \neq i_q \geq 1}^{2p-1} i_1 i_2 i_3 \cdots i_q \tag{15}$$

and for any X,

$$\sum_{p=0}^{\infty} \frac{p^S}{p!} X^S = (\sum_{n=1}^{S} \mathrm{a}_{s,\,n} X^n)\, \mathrm{e}^X \tag{16}$$

where

$$\begin{aligned} \mathrm{a}_{s,\,s} &= 1 \\ \mathrm{a}_{s,\,n} &= \sum_{k_1 \geq k_2 \geq k_3 \cdots k_{s-n} \geq 1}^{n} k_1 k_2 k_3 \cdots k_{s-n} \qquad (\text{for } n < s) \end{aligned} \tag{17}$$

Now we rewrite eq.(11) as

$$\Delta(\hbar) = ln[1 + \mathrm{e}^{\frac{X}{\hbar}}\, \bar{\delta}(X, Y)] \tag{18}$$

with

$$\bar{\delta}(X, Y) = \sum_{m=1}^{\infty} \sum_{p=1}^{m-1} (-\hbar)^{p-m} \frac{(3m!)}{m!\,(3m-2p)!\,2^p\,p!} X^m Y^p \tag{19}$$

and

$$X \equiv \frac{c}{3!}(a^{-1}b)^3 \ , Y \equiv a^{-1}(a^{-1}b)^{-2} \tag{20}$$

In eq.(19) we retained only terms which do not vanish in the limit $\hbar \to 0$. Using eq.(15) in eq.(19) one finds readily

$$\sum_{p=1}^{\infty}\sum_{q=0}^{2p-1}\sum_{m=p+1}^{\infty}(-\hbar)^{p-m}\,\frac{(-1)^q\,3^{2p-q}}{2^p\,p!\,q!}\,\frac{m^{2p-q}}{m!}\,X^m\,Y^p \sum_{i_1\neq i_2\neq i_3\cdots\neq i_q\geq 1}^{2p-1} i_1 i_2 i_3\cdots i_q \tag{21}$$

Now we make use of eq.(16) in the form

$$\sum_{m=p+1}^{\infty}\frac{m^{2p-q}}{m!}\,X^m = (\sum_{n=1}^{2p-q} \mathrm{a}_{2p-q,n}\,X^n)\,\mathrm{e}^X - \sum_{m=1}^{p}\frac{m^{2p-q}}{m!}\,X^m$$

to write

$$\bar{\delta}(X,Y) = \mathrm{e}^{\frac{-X}{\hbar}} \sum_{p=1}^{\infty}\sum_{q=0}^{p-1}\sum_{n=p+1}^{2p-q}(-\hbar)^{p-n}\,\frac{(-1)^q 3^{2p-q}}{2^p\,p!\,q!}\,X^n\,Y^p\,\mathrm{a}_{2p-q,n}\sum_{i_1\neq i_2\cdots\neq i_q\geq 1}^{2p-1} i_1 i_2\cdots i_q \tag{22}$$

In the above equation we again dropped out terms which are irrelevant in the limit $\hbar \to 0$ and thus made consistently the replacement

$$\sum_{p=1}^{\infty}\sum_{q=0}^{2p-1}\sum_{n=1}^{2p-q} \to \sum_{p=1}^{\infty}\sum_{q=0}^{p-1}\sum_{n=p+1}^{2p-q}.$$

We are thus lead to the crucial fact that the exponential suppression in the argument of the log in eq.(11), will be fully compensated by the exponential factor spelled out in eq.(22). One still has to study the behavior of the remaining power series in $\hbar$. After a trivial change of variable in the summation indices one gets for $\Delta(\hbar)$

$$\Delta(\hbar) = ln[1 + \sum_{l=1}^{\infty}\sum_{r=0}^{\infty}(-1)^l\Big(\frac{9X^2Y}{2\hbar}\Big)^l\,\Big(\frac{9XY}{2}\Big)^r\ \mathrm{b}_{l,r}] \tag{23}$$

where $\mathrm{b}_{l,r}$ denotes the following complicated but well defined expression

$$\frac{1}{(l+r)!}\sum_{q=0}^{r}\frac{(-1)^q}{3^q}\,(\sum_{i_1>i_2>i_3>\cdots i_q=1}^{2l+2r-1} i_1 i_2 i_3\cdots i_q\,)\,(\sum_{k_1\geq k_2\geq k_3\cdots k_{r-q}\geq 1}^{2l+r} k_1 k_2 k_3\cdots k_{r-q}\,) \tag{24}$$

Fortunately all what we need to remember from the above intricacy is that the dependence in $\hbar$ comes exclusively in the summation over l, *i.e.* no $\hbar$ to the power r and no $\hbar$ dependence in $\mathrm{b}_{l,r}$. It then follows that the argument of the log in eq.(23)

behaves like $e^{-9X^2Y/2\hbar}$ in the quasistatic limit, i.e. $\dot{q}$ sufficiently small. To see this we note that

$$1+\sum_{l=1}^{\infty}\sum_{r=0}^{\infty}\left(\frac{-9X^2Y}{2\hbar}\right)^l\left(\frac{9XY}{2}\right)^r \mathrm{b}_{l,r} \;=\; e^{-9X^2Y/2\hbar}+\sum_{l=1}^{\infty}\sum_{r=1}^{\infty}\left(\frac{-9X^2Y}{2\hbar}\right)^l\left(\frac{9XY}{2}\right)^r \mathrm{b}_{l,r} \tag{25}$$

Now it is clear from eq.(20) that XY depends linearly on $\dot{q}$ and thus $\left(\frac{9XY}{2}\right)^r \mathrm{b}_{l,r}$ can be made arbitrarily smaller than 1 for sufficiently small $\dot{q}$. In this limit the leading term in eq.(25) is the exponential and one gets

$$\hbar\Delta(\hbar) \;\sim\; \frac{9}{2}X^2Y \tag{26}$$

which means that the effective lagrangian in eq.(13) is indeed plagued with an induced ($\hbar$ independent) classical contribution which was not present in the original classical lagrangian eq.(4) (in the case $n \leq 3$). It is now clear from eqs.(12, 20, 26) that *the full correspondence between the classical and quantum system will be restored only if* $f_3(q) = 0$ *for all* q.

This ends our proof. The generalization to the case $n \geq 3$ goes along the same lines, albeit further technical intricacies, and so will not be persued here[3]. The general constraint is thus

$$f_n(q) = 0 \quad (n \geq 3). \tag{27}$$

In the next section we discuss some of the possible implications of the above result on issues relating to low energy effective lagrangians in particle physics, and more specifically to electroweak interactions.

3 Comments and physical implications

First, it is useful to note that the constraint eq.(27), can be avoided (at least as far as the correspondence principle is concerned) if the $f_n(q)$, $(n \geq 3)$ are themselves quantum effects, that is if they depend on $\hbar$ and vanish in the classical limit. This is typically what happens when the higher derivative interactions are induced by

[3]details will be given in the appendix of ref.[14]

perturbative loop corrections. Thus our result can be rephrased as follows : *if one insists on having higher derivative interactions, then these are bound to be quantum corrections.*

All the above results can be carried over to relativistic field theory almost straighforwardly, at least in the case of scalar fields to start with[4]. Supplemented by the requirement of Lorentz invariance one has from eq.(27) (in the case of neutral scalar fields), that all operators of the form

$$\partial^{\mu_1}\phi\,\partial_{\mu_1}\phi\;\partial^{\mu_2}\phi\,\partial_{\mu_2}\phi\cdots\partial^{\mu_n}\phi\,\partial_{\mu_n}\phi \qquad \text{with} \quad n\geq 2$$

should be either absent or induced by radiative corrections. In the issue this seems to tie up nicely with familiar considerations related to renormalizability and the classification of operators into relevant, marginal and irrelevant in the infrared regime [4].

Yet one should stress two main differences. In deriving the usual operator flow equations [4], the crucial assumption is that the new physics is characterized by a sufficiently high energy scale so that the infrared observables become insensitive to almost any change in this scale. In the present approach no specific reference whatsoever, to any underlying physics is made, if not simply the general motivation for studying higher derivative interactions. The second point is that the flow equations analysis applies to any type of operators (the main issue being the dimension of the operators)[4], while in the present case only those with temporal derivatives are concerned.

Then it should be clear that the resulting constraints will apply even if the low energy phenomena were sensitive to a (near) new physics scale, and will thus lead to complementary restrictions as regards the relevance of higher dimensional operators. Specific examples will be given below. But before doing so, a further comment is perhaps useful at this stage, concerning the case when the higher derivative operators are actually induced by quantum effects. Even in this case one might still want the ensuing interactions to be strictly vanishing when vector fields are invovled, in order to

[4]where now the dot in eq.(4) refers only to the temporal derivative.

preserve the Lorentz invariance in the low energy regime. Indeed if eq.(4) denotes now a *Lorentz invariant* Lagrange density for an interacting covariant vector field ($q \Rightarrow W_\mu$ and $\dot{q} \Rightarrow \partial_0 W_\mu$) , then $\hbar\Delta(\hbar)$ is generally not a Lorentz invariant, as can be inferred from the structure of eqs.(26, 20, 12) and should be formally required to vanish. In fact this feature constitutes the full generalization of previous investigations where only the structure of the determinant in eq.(10) was considered [5,6]. So in some sense the analysis in this paper addresses the question of consistent quantizability, from the point of view of the correspondence between classical and quantum systems or/and the preservation of space–time symmetries at the quantum level.

We turn now to the subject of effective interactions in the context of electroweak theory. This is of relevance to various phenomenological studies carried out in the past few years [7], to assess the ability of future colliders in testing the gauge couplings among the electroweak vector bosons as predicted by the standard model [8]. It also lead to some controversy [7, 9–11] related to whether it is at all theoretically reasonable to expect "big" deviations away from the gauge couplings as a sign for "beyond" the standard model. Here we do not intend to discuss this very extensively. We simply note that, stripped to its essence, the answer to these questions depends ultimately on whether the supposed "new physics" lies at relatively low energy (comparable to the electroweak scale) or at scales of the order of the Tev (or maybe $\sim 10^{15}$ Gev).

Hereafter we want to show briefly how the analysis performed in this paper can be used to give indirect hints about the possible physical origin and expected orders of magnitude of the various effective interactions studied in the literature. As an illustration we consider the following higher dimensional operators taken from ref.[10].

$$O_{B\Phi} = iB^{\mu\nu}(D_\mu\Phi)^\dagger D_\nu\Phi \tag{28}$$

$$O_{W\Phi} = i(D_\mu\Phi)^\dagger \; \vec{\tau} \, . \, \vec{W}^{\mu\nu} \; D_\nu\Phi \tag{29}$$

$$O'_{W\Phi} = i(\Phi^\dagger \; \vec{\tau} \, . \, \vec{W}^{\mu\nu} \; \Phi)(D_\mu\Phi)^\dagger D_\nu\Phi \tag{30}$$

$$O^{(1)}_{\gamma\Phi} = \Box(\Phi^\dagger \; \vec{\tau} \, . \, \vec{W}^{\mu\nu} \; \Phi)D_\mu\Phi^\dagger D_\nu\Phi \tag{31}$$

$$O^{(3)}_{\gamma\Phi} = \partial_\rho(\Phi^\dagger \; \vec{\tau} \, . \, \vec{W}^{\mu\nu})(D^\mu\Phi^\dagger \; \vec{\tau} \, . \, \vec{W}^{\mu\nu} \; \Phi + \Phi^\dagger \; \vec{\tau} \, . \, \vec{W}^{\mu\nu} \; D^\mu\Phi) \tag{32}$$

$$
\begin{aligned}
\hat{\hat{O}}_Z &= D_\mu((D_\nu\Phi^\dagger)\Phi - \Phi^\dagger D_\nu\Phi)\,[\,(D^\mu\Phi^\dagger)D^\nu\Phi + D^\nu\Phi^\dagger D^\mu\Phi \\
&\quad + \frac{1}{<\Phi>^2}((D^\mu\Phi^\dagger)\Phi - \Phi^\dagger D^\mu\Phi)((D^\nu\Phi^\dagger)\Phi - \Phi^\dagger D^\nu\Phi)\,] \qquad (33)\\
\hat{\hat{O}}_\gamma^{(1)} &= D_\mu D^\rho(\Phi^\dagger\,\vec{\tau}\,.\,\vec{W}^{\rho\nu}\;\Phi)(D^\mu\Phi^\dagger D^\nu\Phi + D^\nu\Phi^\dagger D^\mu\Phi) \qquad (34)\\
\hat{\hat{O}}_\gamma^{(2)} &= D_\mu D^\rho(\Phi^\dagger\,\vec{\tau}\,.\,\vec{W}^{\rho\nu}\;\Phi)((D^\mu\Phi^\dagger)\Phi - \Phi^\dagger D^\mu\Phi)((D^\nu\Phi^\dagger)\Phi - \Phi^\dagger D^\nu\Phi) \qquad (35)
\end{aligned}
$$

where Φ denotes the usual Higgs doublet and $\vec{W}_\mu$ and B_μ are the $SU_L(2)$ and $U_Y(1)$ gauge bosons. The above operators have been studied in [10] as an illustration of the possibility to generate anomalous W couplings from gauge invariant higher dimensional interactions in the context of an effective spontaneously broken model [12]. The question however, is to know whether these effects do not turn out to be, after all, of the order of perturbatively small radiative corrections, or even simply a reformulation of the standard radiative corrections themselves, which generate small anomalous couplings. To avoid this situation, one can try to interprete the operators above as originating from quantum non–perturbative effects. This could be achieved *a priori* by assuming for instance multi–fermion interactions at the underlying level, and making use of general equivalence conditions *à la* Lurié–Macfarlane [13]. It is interesting to note that in this case the induced interactions become classical (at least in the leading $1/N$ expansion) in the sense that $\hbar$ powers cancel out completely from the vertices, once the poles in the W and Φ propagators are properly identified. In such a non–perturbative scenario the magnitude of the induced interactions might even be large.

Nevertheless the constraint eq.(29), can rule out this possiblity for a certain type of operators. For instance $O_{\gamma\Phi}^{(1)}$ defined previously, has a contribution with 3 time–derivatives of Φ and should thus be suppressed according to eq.(29) [5]. An immediate consequence is that a large departure (δ_γ) from the γW^+W^- Yang–Mills coupling will

[5] Note that eq.(29) applies directly to this case even if Φ is complex, since Φ will anyway induce a neutral physical scalar.

be difficult to induce from the gauge invariant combination

$$O_{\gamma\Phi} = \frac{ie}{M_W^2} \left(\frac{O_{\gamma\Phi}^{(1)}}{< \Phi >^2} + \cdots \right) \tag{36}$$

suggested in ref.[10], while small quantum effects are still allowed. The same conclusion holds for $\hat{\hat{O}}_Z, \hat{\hat{O}}_\gamma^{(1)}$ and $\hat{\hat{O}}_\gamma^{(2)}$. Thus the CP–violating (C–violating, P–conserving) anomalous ZWW and γWW couplings, obtained respectively from $\hat{\hat{O}}_Z$ and a special combination of $\hat{\hat{O}}_\gamma^{(1)}, \hat{\hat{O}}_\gamma^{(2)}$ (and two other operators, see [10] for details), should be vanishing or at most perturbatively small. This is in accordance with what one would naively expect for CP–violating effects in the bosonic sector.

In contrast, the level to which we carried out the analysis in this paper does not yet allow to draw definite conlusions concerning other operators like $O_{B\Phi}, O_{W\Phi}, O'_{W\Phi}$, $O_{\gamma\Phi}^{(3)}$ ... etc. As stated before, in this case one has to take into account the quantization of spin 1 (massive) fields, which generally brings up additional constraints [6], related to the Loretnz invariance of the measure in the path itegral. This is now under investigation.

4 Conclusion

To summarize, we have worked out in this paper the general path integral quantization of higher derivative interactions in the simplest quantum mechanical case. As far as we know this has never been treated before beyond the quadratic case [3]. Stringent consistency requirements related to the correspondence principle have thus been identified, at least in the quasistatic limit and used to gain more insight in the possible origin of higher dimensional effective operators, involving scalar and vector fields in the context of electroweak interactions.

The approach is however readily generalizable to a variety of other physical situations, and can be helpful in understanding the interplay between physics at different energy scales.

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